Properties of Odd-frequency Superconductivity in Antiferromagnetic Ordered State

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We investigate properties below $T_{\rm C}$ of odd-frequency pairing which is realized by antiferromagnetic critical spin fluctuations or spin wave modes. It is shown that $\Delta(\epsilon_n)$ becomes maximum at finite ϵ_n, and $\Delta(\pi T)$ becomes maximum at finite T. Implications of the present results to the experimental results of $CeCu_2Si_2$ and $CeRhIn_5$ are given.

KEYWORDS: odd-frequency, gapless superconductivity, coexistence of antiferromagnetism and superconductivity, $CeCu_2Si_2$, $CeRhIn_5$

1. Introduction

In Ce-based heavy fermion compounds, it is known that $CeCu_2Si_2$ and $CeRhIn_5$, under pressure, exhibit two kinds of superconductivity (SC), gapless SC and line-node gap SC, from measurements of the nuclear spin-lattice relaxation rate $1/T_1$ [1–3]. At low pressure side of the ”critical pressure”, antiferromagnetism (AF) and the gapless SC coexist below the superconducting transition temperature $T_{\rm C}$. On the other hand, at high pressures, AF disappears and the line-node SC appears. This gapless SC is not due to impurity scatterings, because the clear line-node gap SC recovers with the same sample at pressures exceeding the critical pressure. Namely, the gapless SC seems to be an intrinsic and a novel SC state.

It was pointed out that the gapless nature can be understood as the odd-frequency p-wave singlet pairing is occurring by critical spin fluctuations and antiferromagnetic spin waves [4]. However, previous theories discussed only a behavior of $T_{\rm C}$. Properties of SC below $T_{\rm C}$ have not been understood yet. In this paper, we investigate these properties below $T_{\rm C}$ by using the same spin fluctuation modes as used in Ref. 4. In particular, we examine property of the frequency dependent gap function $\Delta(\epsilon_n)$, ϵ_n being the fermionic Matsubara frequency, by solving the gap equation. It is shown that $\Delta(\pi T)$ takes maximum at finite temperature and decreases towards zero as temperature decreases. When the effect of spin fluctuations is not strong, the odd-frequency SC exhibits the reentrant behavior. On the other hand, the odd-frequency SC remains even at zero temperature when the effect of critical spin fluctuation is strong enough at the criticality or in the ordered state of AF.

2. Theory

We introduce the pairing interaction mediated by critical antiferromagnetic spin fluctuations as follows:

$$V(\mathbf{q},\omega_m) = g^2\chi(\mathbf{q},\omega_m) = \frac{g^2 N_{\rm F}}{\eta + A\hat{\mathbf{q}}^2 + C|\omega_m|}, \tag{1}$$

where g is the coupling constant, $N_{\rm F}$ the density of states at the Fermi level, and $\hat{\mathbf{q}}^2 \equiv 4 + 2(\cos q_x + \cos q_y)$ in two dimensions. This type of pairing interaction was adopted by Monthoux and Lonzarich

to discuss the strong coupling effect on the superconducticity induced by the critical AF fluctuations [5]. The parameter η in eq. (1) parameterizes a distance from the QCP. We can treat coexistence phase by setting $\eta = 0$.

The pairing interaction can be decomposed as

$$V_\ell(\epsilon_n - \epsilon_{n'}) = \sum_{\mathbf{k},\mathbf{k}'} \phi_\ell(\mathbf{k}) V_\ell(\mathbf{k}-\mathbf{k}', \epsilon_n - \epsilon_{n'}) \phi_\ell^*(\mathbf{k}') \simeq v_\ell \ln \frac{\epsilon_F}{\sqrt{(\epsilon_n - \epsilon_{n'})^2 + \eta^2}}, \tag{2}$$

where ϵ_n and $\epsilon_{n'}$ are Matsubara frequencies. The interaction, eq. (2), exhibits logarithmic divergence when $\epsilon_n - \epsilon_{n'} \simeq 0$ at the criticality $\eta = 0$. Sharper the divergence is obtained by using smaller the value of the paramter η. We introduce a cutoff ϵ_F and restrict Matsubara frequencies such that $(\epsilon_n - \epsilon_{n'})_{\max} \leq \epsilon_F$. The coupling constant v_ℓ is a positive coefficient which is determined by the relation between the Fermi surface and AF ordering vector. ℓ takes even or odd integer indicating type of paring. When AF ordering vector is comparable to a diameter of the Fermi surface without nesting tendency, p-wave ($\ell = 1$) odd-frequency pairing is promoted against d-wave ($\ell = 2$) even-parity one. In this paper, we suppose v_o is larger than v_e, since we consider the situation that the odd-frequency is promoted by AF background [6].

The gap equation (non linearized) is given as

$$\Delta_{e,o}(\epsilon_n) = -k_B T \sum_{n',k'} V_{e,o}(\epsilon_n - \epsilon_{n'}) \frac{\Delta_{e,o}(\epsilon_{n'})}{\epsilon_{n'}{}^2 + \xi_{k'}{}^2 + |\Delta_{e,o}(\epsilon_{n'})|^2}, \tag{3}$$

where $\xi_{k'}$ is quasiparticle energy measured from the chemical potential (Fermi level). We solved this gap equation selfconsistently.

3. Result

3.1 Frequency dependence of gap function

The frequency dependence of gap function is shown in Fig.1 for $T/T_C = 0.99,\ 0.25,\ 0.01$. Near T_C, the relation $\Delta_o(\epsilon_n) \propto 1/\epsilon_n$ roughly holds. $\Delta_o(\epsilon_n)$ takes maximum at the lowest frequency $\epsilon_0 = \pi T$ as in the ordinary even-frequency SC. On the other hand, at low tmperature $T/T_C = 0.01$, $\Delta_o(\epsilon_n)$ takes maximum at finite ϵ_n, reflecting the odd frequency nature, while the relation $\Delta_e(\epsilon_n) \propto 1/\epsilon_n$ roughly holds in the whole temperature region in the case of even-frequency SC.

3.2 Temperature dependence of gap function

The temperature dependence of gap function is shown in Fig.2 for $\eta = 0.008,\ 0.005,\ 0.001$. Solid lines represent $\Delta_o(\pi T)$. Broken lines represent $\Delta_{o\,\max}(\epsilon_n)$. $\Delta_{o\,\max}(\epsilon_n)$ coincides with $\Delta_o(\pi T)$ in high temperature region. However, $\Delta_{o\,\max}(\epsilon_n)$ deviates from $\Delta_o(\pi T)$ in low temperature region. This difference arises from reduction of gap function in low frequency region, as shown in Fig.1.

$\Delta_o(\pi T)$ takes a maximum at around $T \simeq T_C/2$ and $\Delta_o(\pi T)$ decreases as temperature decreases. This reduction is in contrast to the case of ordinary even-frequency gap, in which $\Delta_e(\pi T)$ saturates at low temperatures. When $\eta = 0.008$, $\Delta_o(\pi T)$ vanishes at low temperature side, exhibiting reentrant behavior. Whereas for $\eta = 0.005,\ 0.001$ and $\Delta_o(\pi T)$ is finite in the whole region of $T \leq T_C$. Maximum at finite T of $\Delta_o(\pi T)$ is due to the maximum at finite ϵ_n of $\Delta_o(\epsilon_n)$ at low temperatures.

4. Conclusion

We have solved gap equation for odd-frequency pairing realized by the antiferromagnetic critical spin fluctuations or spin wave modes of AF. The gap function of odd-frequency SC takes maximum with respect to ϵ_n and T. Reentrant behavior occurs when η is not extremely small. On the other hand, the odd-frequency SC is realized when η approaches zero. Thus, the coexistence phase of SC and AF

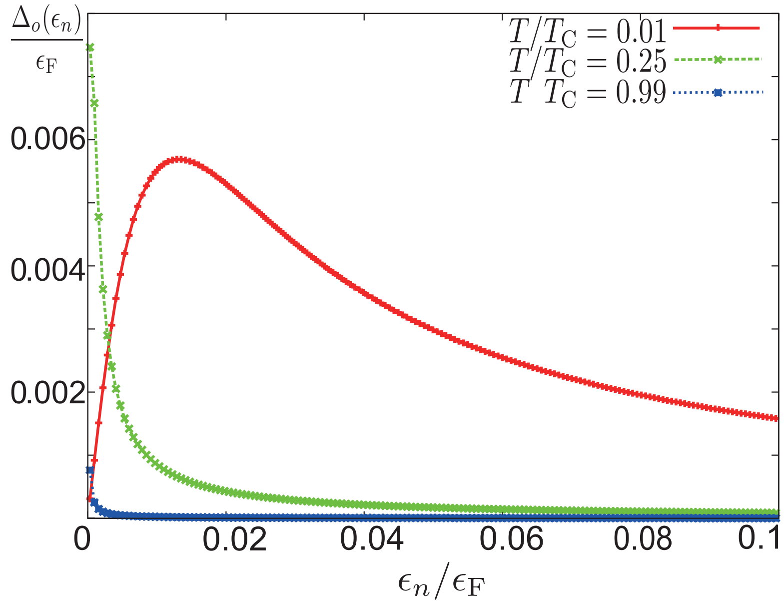

Fig. 1. Frequency dependences of gap function in the case of $\eta = 0.005$ for a series of temperatures, T/T_C=0.01, 0.25, and 0.99.

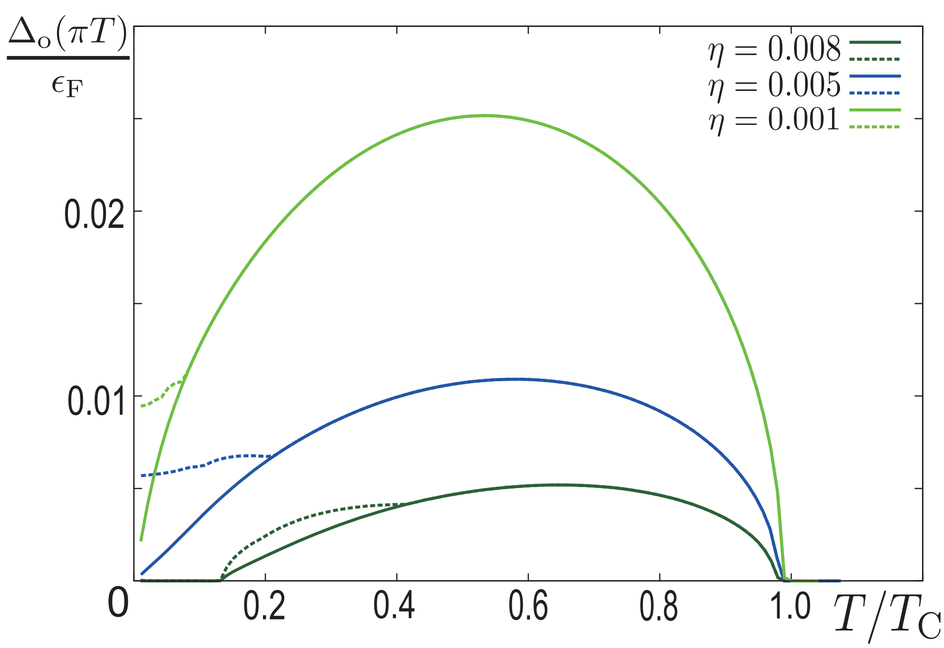

Fig. 2. Temperature dependence of gap function $\Delta_o(\pi T)$. Broken lines represent $\Delta_{o\,max}(\epsilon_n)$.

observed in $CeCu_2Si_2$ and $CeRhIn_5$ can be simulated by our model with zero η limit. In this situation, $\Delta_o(\pi T)$ is finite even at $T = 0$.

At very low temperature region, however, odd-frequency can compete with even-frequency. If we consider this competition, transition from odd-frequency to even-frequency is possible at very low temperature. This problem will be discussed elsewhere.

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